Cosmological Quantum Jump Dynamics

II. The Retrodictive Universe

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Abstract

This paper is a continuation of the paper [1] and is dedicated to the problem of the arrow of time. A deterministic past-directed dynamics is constructed, which results in the retrodictive universe. A future-directed dynamics of the latter is indeterministic and reproduces standard probabilistic quantum dynamics. The arrow of time is inherent in the retrodictive universe as well as a future-directed increase of informational entropy.

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Introduction

One of the most ancient and difficult problems of physics is that of the origin of the arrow of time, i.e., of the nature of the difference between the past and the future. It is conventional to search for a solution to this problem in dynamics, i.e., time evolution of a state of a physical system. The solution may be given by a dynamics which is asymmetric, or orientable in the sense of the sequence of states.

A deterministic dynamics does not involve such an orientability. Therefore it seems advisable to search for the solution in an indeterministic dynamics. But, according to an established opinion, in the standard indeterministic quantum dynamics there is no arrow of time as well [2,3].

A crucial feature of an orientable dynamics is the prevalence of retrodiction over prediction. An essential mathematical concept inherent in indeterminism is that of randomness [4-6]. It is the latter that seems to be the main impediment to the construction of a dynamics with the above-mentioned feature.

The simplest way of constructing an orientable dynamics is to construct one in which a past-directed time evolution would have been deterministic. In doing this, the pivotal point is the condition that a future-directed time evolution represent the standard indeterministic, i.e., probabilistic quantum dynamics.

In the present paper, that idea is realized. We introduce a jump-deciding mechanism for a two-state tangency vertex. The mechanism consists of the Planck clock (one with the Planck time period) and rules which decide a quantum jump by the reading of the clock. On the basis of this mechanism, a deterministic past-directed dynamics is constructed. The corresponding future-directed dynamics turns out to be indeterministic. The rules are selected in such a way that probabilities be standard quantum ones.

The construction outlined above exhibits a retrodictive universe. Its salient features are the following. A complete retrodiction and a partial prediction are involved. It is impossible to introduce initial conditions and to extend dynamics forward in time; thus the retrodictive universe is constructed at once as a whole—for all times from a maximal future to a maximal past. In this connection we quote Weyl [7]: "The objective world simply is, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time."

1 The problems of the arrow of time and randomness

1.1 The problem of the time arrow

We quote Roger Newton [8]: "One of the greatest puzzles of physics is the manifest discord between two facts: on one hand, all the fundamental equations and laws of physics are (essentially) invariant under time reversal;. . . on the other hand, we are all aware that at the macroscopic level, many physical processes flow in one time direction only. The unidirectional flow of time . . . is one of the most obvious features both of our consciousness and of the physical world, . . . which near the end of the nineteenth century presented physics with one of the

most profound challenges, and about which there are, to this day, strong disagreements among physicists."

There are five arrows of time [8]:

1. The delay between cause and effect.
2. The biological, or *cognitive* arrow.
3. The second law of thermodynamics.
4. The cosmological arrow (the expansion of the universe).
5. The direction of the time parameter used in physics.

The fifth arrow is present in any dynamics. The second and third arrows are related by an informational aspect: information on the past is greater than on the future, so that (informational) entropy increases in time. It is this aspect that forms the basis for our attacking the time arrow problem.

1.2 The problem of randomness

The mathematical concept of randomness is inherent in an indeterministic, i.e., probabilistic dynamics. A physical problem related to the concept is the impossibility of empirically verifying or falsifying the randomness of results of a generic quantum dynamical process. We quote Beltrami [4]: "...there can be *no formal proof that a sufficiently long string is random* ... In response to the persistent question '*Is it random?*' the answer must now be 'probably, but I'm not sure'; 'probably' because most numbers are, in fact, random ... and 'not sure' because the randomness of long strings is essentially undecidable." Thus the question of the randomness of quantum dynamics cannot be, strictly speaking, decided empirically. So there remains the possibility of choosing a decision which would provide a resolution of some theoretical problems.

1.3 Time nonorientability of the standard indeterministic quantum dynamics

There exists no arrow of time in the standard indeterministic quantum dynamics. This is exhibited by means of a fully time-symmetric construction from which conventional quantum mechanics may be derived [9] (see a detailed treatment in [3]).

Since the conventional indeterministic (probabilistic) dynamics is based on the concept of randomness, it seems reasonable to try to abandon the latter in constructing an orientable dynamics: we do not see what else may be done.

2 A deterministic past-directed quantum jump dynamics

2.1 Tangency vertex

Let us consider a tangency vertex [1], i.e., a confluence or/and branch point of levels without crossing, in more detail. Assume that without quantum jumps the Hamiltonian $H(t)$ is a C^∞

operator-valued function of time, so that for a vertex

$$\frac{d^n H_{\text{ver}}^-}{dt^n} = \frac{d^n H_{\text{ver}}^+}{dt^n} = \frac{d^n H_{\text{ver}}}{dt^n}, \quad n = 0, 1, 2, \ldots \tag{2.1.1}$$

holds. In an infinitesimal neighborhood of the vertex,

$$H(t_{\text{ver}} + \Delta t) = H[g(t_{\text{ver}} + \Delta t)] = H[g(t_{\text{ver}}) + \dot{g}(t_{\text{ver}})\Delta t] \tag{2.1.2}$$

and

$$H_{\text{ver}}(t_{\text{ver}} \mp \Delta t) = \sum_l^{1,n^\mp} \varepsilon_l(t_{\text{ver}} \mp \Delta t) P_l(t_{\text{ver}} \mp \Delta t), \quad \Delta t \geq 0 \tag{2.1.3}$$

For any projector $E(t)$

$$E \frac{dE}{dt} E = 0 \tag{2.1.4}$$

is fulfilled, so that

$$\frac{d^n}{dt^n} \left(E \frac{dE}{dt} E \right) = 0, \quad n = 0, 1, 2, \ldots \tag{2.1.5}$$

Hence it is easily seen that from

$$\frac{d^m E^-}{dt^m} = \frac{d^m E^+}{dt^m} \qquad \text{for} \quad m = 0, 1, \ldots, n \tag{2.1.6}$$

follows

$$\left(E \frac{d^{n+1} E}{dt^{n+1}} E \right)^- = \left(E \frac{d^{n+1} E}{dt^{n+1}} E \right)^+ \tag{2.1.7}$$

Now for a tangency vertex, we obtain by the method of [1]

$$\frac{d^n P_{\tan}^-}{dt^n} = \frac{d^n P_{\tan}^+}{dt^n} = \frac{d^n P_{\tan}}{dt^n}, \qquad n = 0, 1, 2, \ldots \tag{2.1.8}$$

where

$$P_{\tan}^\mp = \sum_l P_l^\mp = P_{\tan} \tag{2.1.9}$$

and

$$\frac{d^n \varepsilon_l^-}{dt^n} = \frac{d^n \varepsilon_{l'}^+}{dt^n} = \frac{d^n \varepsilon_{\tan}}{dt^n}, \qquad n = 0, 1, 2, \ldots \tag{2.1.10}$$

Thus, at a tangency vertex, there is contact of order $n = \infty$.

2.2 A two-state jump-deciding mechanism in a past-directed dynamics

We assume that the degeneracy of levels occurs only at vertices, so that there are only two-state vertices. Therefore we consider a two-state tangency vertex. For the latter, we introduce a jump-deciding mechanism. The mechanism consists of a Planck clock and rules deciding a jump by the reading of the clock. The Planck clock is one with the Planck time t_{P} period.

(There is no other natural time interval.) We will measure time in units of $t_{\rm P}$ so that the period of the clock is 1.

There are two states $i = 1, 2$ for $t < t_{\rm tan}$ and two states $f = 1, 2$ for $t > t_{\rm tan}$. We consider transitions, i.e., quantum jumps $i \to f$ in a future-directed dynamics and transitions $f \to i$ in a past-directed dynamics. It is the latter that has to be deterministic, so that let $f = a$ be an actual state before a quantum jump $a \to i$. Quantum probabilities are

$$p(f|i) = p_{f\to i} = p_{i\to f} = |\langle f|i\rangle|^2 \tag{2.2.1}$$

with

$$\sum_f p(f|i) = \sum_i p(f|i) = 1 \tag{2.2.2}$$

Let

$$\epsilon_{i=1} < \varepsilon_{i=2} \tag{2.2.3}$$

Introduce some $\bar t$,

$$0 \le \bar t \le 1 \tag{2.2.4}$$

A deterministic past-directed dynamics is defined by the following jump-deciding rule for $a \to i$:

$$\text{if} \quad 0 \le t_{\rm tan} < \bar t \quad \text{then} \quad a \to 1, \qquad \text{if} \quad \bar t \le t_{\rm tan} < 1 \quad \text{then} \quad a \to 2 \tag{2.2.5}$$

2.3 A future-directed dynamics

In order to provide a correct probabilistic future-directed dynamics, we introduce

$$p_{\rm min/max} = ({\rm min/max})_i\{p(a|i)\}, \quad p_{\rm min} \le 1/2 \le p_{\rm max} \tag{2.3.1}$$

$$t_1 = \frac{p_{\rm min}}{1/2 + p_{\rm min}}, \quad t_2 = \frac{1/2}{1/2 + p_{\rm min}}, \quad t_1 \le t_2, \quad t_1 + t_2 = 1 \tag{2.3.2}$$

and put

$$\bar t = t_i \quad \text{where} \quad p(a|i) = p_{\rm min} \tag{2.3.3}$$

The rule (2.2.5) is equivalent to these:

$$\text{if} \quad 0 \le t_{\rm tan} < t_1 \quad \text{then} \quad a \to 1, \qquad \text{if} \quad t_2 \le t_{\rm tan} < 1 \quad \text{then} \quad a \to 2 \tag{2.3.4}$$

$$\text{if} \quad t_1 \le t_{\rm tan} < t_2 \quad \text{then} \quad a \to i \quad \text{where} \quad p(a|i) = p_{\rm max} > 1/2 \quad (\text{here} \quad t_1 < t_2) \tag{2.3.5}$$

Now if an initial state is i, then, in view of (2.3.4),

$$t_{i-1} \le t_{\rm tan} < t_{i+1} \tag{2.3.6}$$

where

$$0 = t_0 \le t_1 \le t_2 \le t_3 = 1 \tag{2.3.7}$$

and

$$t_{i+1} - t_{i-1} = t_2 \tag{2.3.8}$$

Therefore we should have for probabilities

$$p_{\rm min} = \frac{t_1/2}{t_2}, \quad p_{\rm max} = \frac{(t_2 - t_1) + t_1/2}{t_2} = \frac{t_2 - t_1/2}{t_2} \tag{2.3.9}$$

since $t_1 \le t_{\rm tan} < t_2$ implies $p(a|i) = p_{\rm max}$, whereas if $t_{\rm tan} < t_1$ or $t_{\rm tan} \ge t_2$ then $p(a|i) = p_{\rm min}$ and $p(a|i) = p_{\rm max}$ are equiprobable. The choice (2.3.3) provides (2.3.9).

2.4 The Planck clock and two fundamental theories

For the sake of generality, we might choose any time period $t_{\rm per}$ rather than the Planck time $t_{\rm P}$. We have put

$$t_{\rm per} = t_{\rm P} \tag{2.4.1}$$

for lack of any other natural time interval. There is an added reason for incorporating the Planck clock into quantum jump dynamics, i.e., putting (2.4.1).

The Planck time

$$t_{\rm P} = \left(\frac{\hbar G}{c^5}\right)^{1/2} = 5.3906 \times 10^{-44}{\rm s} \tag{2.4.2}$$

may be introduced as a natural unit for all physical quantities. Put

$$c = 1 \tag{2.4.3}$$

then

$$\hbar G = t_{\rm P}^2 \tag{2.4.4}$$

Next, there are two systems of units:

$$\hbar = 1, \quad G = t_{\rm P}^2 \tag{2.4.5}$$

(natural units) and

$$G = 1, \quad \hbar = t_{\rm P}^2 \tag{2.4.6}$$

("geometrized units" [10]). In the natural units, general relativity involves $t_{\rm P}$: it appears in the Einstein equation, but the standard quantum theory does not involve $t_{\rm P}$: it does not appear in the Schrödinger equation and quantum probabilities. By contrast, in the geometrized units, it is quantum theory rather than general relativity that involves $t_{\rm P}$.

The Planck clock incorporates $t_{\rm P}$ into quantum theory via quantum jump dynamics. Now, in the natural units, general relativity and quantum theory enjoy equal rights with respect to $t_{\rm P}$, which links those fundamental theories in addition to the construction introduced in [1].

3 The retrodictive universe and its salient features

3.1 The retrodictive universe

The construction accomplished above exhibits a universe which is naturally called retrodictive. Let us consider its salient features.

3.2 A complete retrodiction and a partial prediction

Since the past-directed dynamics is deterministic, in the retrodictive universe there exists a complete retrodiction, to which the universe owes its name.

An informational aspect of retrodiction is this: A complete information on the universe's states in the past reduces to and may be obtained from the information contained in the description of a present state and in the laws of the past-directed dynamics.

The future-directed dynamics is indeterministic and reproduces the dynamics of the standard probabilistic quantum theory. Notwithstanding this fact, there exists a partial prediction provided by (2.3.5): if

$$t_1 \leq t_{\tan} < t_2 \tag{3.2.1}$$

then in the future-directed dynamics the jump $i \to f$ happens to the f for which

$$p(f|i) = p_{\max} > 1/2 \tag{3.2.2}$$

But this partial prediction does not violate the probabilistic relations (2.3.9).

3.3 The impossibility of introducing initial conditions

The past-directed dynamics constructed backwards in time starting from some state at $t = t_{\text{initial}}$, $-\infty < t_{\text{initial}} < \infty$, cannot be extended forward in time: we may run into a situation where

$$i = 1, \quad t_2 \leq t_{\tan} < 1 \qquad \text{or} \quad i = 2, \quad 0 \leq t_{\tan} < t_1 \tag{3.3.1}$$

which is inconsistent with the rule (2.3.4). Thus it is impossible to construct a future-directed dynamics starting from some initial conditions. It is only a past-directed dynamics that may be constructed in the case of the retrodictive universe.

3.4 The entirety of the retrodictive universe

The impossibility of constructing a future-directed dynamics starting from some initial conditions given at some t_{initial}, i.e., solving the Cauchy problem, implies the entirety of the retrodictive universe: the latter is determined at once for all times from a maximal future to a maximal past. The universe exists but does not evolve in time [7]. A seeming evolution is the result of the indeterministic character of the future-directed dynamics.

This conclusion cracks the problem of initial conditions for the universe: there are none.

3.5 The arrow of time

The arrow of time is inherent in the retrodictive universe as well as a future-directed increase of informational entropy.

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